Matrices

An m×n matrix A is a rectangular array of mn numbers arranged in m rows and n columns.

A=

ij th componets of A (denoted) is the number appearing in the i th row and j th column of A.

Example A = , B = , C=

An m×n matrix will all componets equal zero is called the m×n zero matrix

Example A =

If A is m×n matrix with m = n then A is called square matrix.

A =

,,,…,,…, form the main diagonal of A.

Example A = , B =

A square matrix is called diagonal matrix if all terms off the main diagonal are zero.

Example A = , B =

A square matrix is called scalar matrix if all terms on the main diagonal are equal.

Example A = , B=

A scalar matrix is called identity matrix if every terms in the main diagonal equal one.

Example A = , B=

A square matrix is called upper triangular if = 0 I j.

A square matrix is called lower triangular if = 0 I j.

Example A= lower, B = upper

Equality of matrices

Let A = , B = be m×n matrices . A is equal to B(denoted by A =B) if = , 1 i m , 1 j n

Example A= , B =

A = B iff x=2 , y = 5 , z = 0 , w = 2

Operation on matrices

1) Addition matrices

Let A = , B = be m×n matrices. The sum of A and B(denoted by A +B) defined to the

matrix C = ,where = + ,1i m ,1j nExample A = , B = , A+B =

2) Scalar multiplication

Let A = be m×n matrix and r is a real number then the scalar multiple of A by r

(denoted by rA) defined to the matrix D = , where = r , 1 I m , 1 j n

Example A = , r = 2, 2A =

3) Matrix multiplication

Let A = be m×p matrix , B = be p×n matrix then the product of A and B (denoted by

AB) defined to be the m×n matrix C = , where = + + … , 1 i m , 1 j n

Example A = , B = , AB =

Let A = be m×p matrix , B = be p×n matrix1) If n m then BA may not be defined

2) If n = m then BA is p×p, while AB is m×m. Thus if p m . AB and BA are different

sizes.

3) AB and BA are both of the same size . But AB BA.

Example A = , B = , AB = , BA is not defined

Example A = , B = , AB =   , BA =

AB BA.

Example A = , B = , AB =   , BA =

Properties of matrix addition

1) A+B =B + A

2) A+(B+C) = (A+B)+C

3)A+0 = 0+A

4) A+(-A) = (-A)+A = 0

Example A = , B = , C = ,

A+B ==B + A

(A+B)+C = + =

A+(B+C) = + =

-A =

A+(-A) = + =

(-A)+A = + =

A+0 = + =

0+A = +=

A-B = A +(-B)

Example A = , B =

-B =

A-B = A +(-B) =

Associative law for matrix multiplication

Let A = be m×n matrix , B = be n×p matrix and C = be p×q matrix, then A(BC) = (AB)C

Example Let A = , B = , C = ,

AB = , (AB)C =

BC = , A(BC) =

Distribution law for matrix multiplication

Let A = be m×n matrix , B = be n×p matrix and C = be n×q matrix, then A(B+C) = AB+AC

Let A = be m×n matrix , B = be m×n matrix and C = be n×q matrix, then (A+B)C = AC+BC

Example Let A = , B = , C = ,

B+C = , A(B+C) =

AB = , AC = , AB+AC =

Example Let A = , B = , C = ,

A+B = , (A+B)C =

AC = , BC =

AB+AC =

If AB = 0 then A0 , B 0

Example Let A = , B = , AB = . But A0 , B 0

If AB = AC , A0 ,then BC

Example Let A = , B = , C =

AB = AC = .But BC

The transpose of matrix

Let A = be m×n matrix the transpose of A (denoted by)

= = , 1 i m , 1 j n

Example A = , =

B = =

C = , =

Properties of transpose

1) = A

2) = +

3) =

4) =

A matrix A is called symmetric if = A

Example A = , B =

A is symmetric since =A

B is not symmetric since B

Properties of symmetric matrix

1) If A is symmetric then is symmetric

2) If A and B are symmetric then A+B is symmetric

3) If A and B are symmetric then AB is symmetric iff AB = BA

A matrix A is called anti symmetric if = -A

Example A = is anti symmetric since = -A

Properties of anti symmetric matrix

1) If A is anti symmetric then is anti symmetric

2) If A and B are anti symmetric then A+B is anti symmetric

3) If A and B are anti symmetric then AB is anti symmetric iff AB = - BA

The inverse of matrix

An n×n matrix is called nonsingular (invertible) if there is n×n matrix B such that AB

= BA =

The matrix B is called an inverse of A(denoted by)

If there exists no matrix B then A is called singular (n0n invertible)

Example Let A = , B = , AB = BA =

B is an inverse of A(Ais non singular)

If a matrix has an inverse then the inverse is unique

Properties of inverse

1) If A is non singular then is non singular and = A

2) If A and B are non singular then AB non singular and =

3) If A is non singular then =

Example Find the inverse of A =

=

=

= 1   ,   = 0

= 0   , = 1

A = -2 , b = 1 , c = 3/2 , d = -1/2

=

Gauss Jordan method

Example A =

[A:] =

Divided first row on 3

Multiply fist row by -2 and add to second row. Multiply fist row by -4 and add to third row

Divided second row on -9

Multiply second row by -6 and add to fist row. Multiply second row by 23 and add to third row

Divided third row on

Multiply third row by -1 and add to fist row. Multiply third row by -1/3 and add to second row =[:]

=

Determinates

If A = ,then = -

Example A = ,then = 10-(-12) = 22

If A = ,then = - +

Example A =

= - + = -69

There are simple method for finding determinates of matrix = + + – (+ +)

Example A =

= 3(2(4) + 5(3)(-1) + 2(4) (2) – ((-1)(2)(2)+2(3)(3)+4(4)(5))= -69

Properties of determinates

1) =

2) =

Example A =

= = 6

Example A = , B =

= -2 , = 5

AB =

= -10 =

There is a rule to finding the inverse of   matrix

A =

=

Example A =

= 1 , =